gence. This situation has been ignored in the past because the equation governing the thermal penetration depth in the case of Fourier diffusion is directly integrable.

The two phase lags, τ_T and τ_q, and, hence, the ratio Z, may be strong functions of temperature, especially for τ_q. With the fundamental framework for the heat balance integral laid down in this work, extensions to accommodate the temperatue-dependent phase lags can readily be made. This will be left for a future communication.

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Analytical Comparison of Constant Area, Adiabatic Tip, Standard Fins, and Heat Pipe Fins

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Introduction

ELECTRONIC equipment, as well as many other applications, need to be kept within certain temperature limits to function properly and safely. Increasing the surface area, and thus, the heat transfer, using fins attached to the high-temperature areas of the devices is a common solution.

To improve heat dissipation without using larger surface areas, fin efficiency must be increased. One way to improve fin efficiency is through higher thermal conductivity fin materials. Heat pipes have effective thermal conductivities that are one to two orders of magnitude higher than solid fins, and can be considered as an effective substitution for solid fins. Another benefit of heat pipes is their inherent fast thermal response time. Peterson[1] shows an example where 20 W of thermal energy is transferred 0.5 m in a device 1.27 cm in diameter. A solid aluminum fin, a solid copper fin, and a heat pipe were compared using Fourier's law. A copper–water heat pipe with a screen wick was used. The temperature differences along the fins were 460, 206, and 6°C, respectively, which demonstrates the much higher effective thermal conductivity in heat pipes. The copper–water heat pipe was also 76 and 93% lighter than the aluminum and copper fins, respectively.

Zhao and Avedisian[2] researched the concept of adding fins to a copper–water heat pipe to increase the heat transfer. Their study consisted of a fixed fin pitch and a fixed fin shape in a forced convection environment. They achieved a heat flux of 80 W/cm^2 and a total power of 800 W for the longest finned heat pipe tested. By comparison, a finned copper rod of the same length only attained 30 W/cm^2 and a total power of 300 W. Particularly interesting data resulted from their tests of shorter heat pipes. In these cases, the solid copper rod performed better than the heat pipe, suggesting that heat pipes may not always be beneficial. The longer the test specimen, the more the heat pipe outperformed the copper rod.

Another study shows a brass fin made from a planar heat pipe with water as the working fluid.[3] The fin was mounted horizontally. They reported a 22% improvement in fin efficiency for this arrangement. This is a significant improvement over standard fins.

The purpose of this study is to derive an expression for a constant area, adiabatic end condition, heat pipe fin efficiency. This will be used to compare the performance of a heat pipe fin with a standard fin.

Heat Pipe Fin Efficiency

Figure 1 illustrates the heat pipe fin being considered. Neglecting radiation, assuming steady state, and assuming that the temperature varies only in the x direction, conservation of energy applied to a differential element of the wall is

$$k(A - A_v)\frac{\partial^2 T}{\partial x^2} - h_0 P_0 (T - T_\infty) - h_i P_i (T - T_v) = 0 \qquad (1)$$

where k is thermal conductivity of the heat pipe wall, A is the cross-sectional area of the heat pipe (wall plus vapor space), A_v is the cross-sectional area of the vapor space, T is the wall temperature, h_0 is the convection heat transfer coefficient on the outside of the heat pipe, P_0 is the outside perimeter of the heat pipe, T_∞ is the temperature surrounding the heat pipe, h_i is the convection heat transfer coefficient on the inside of the

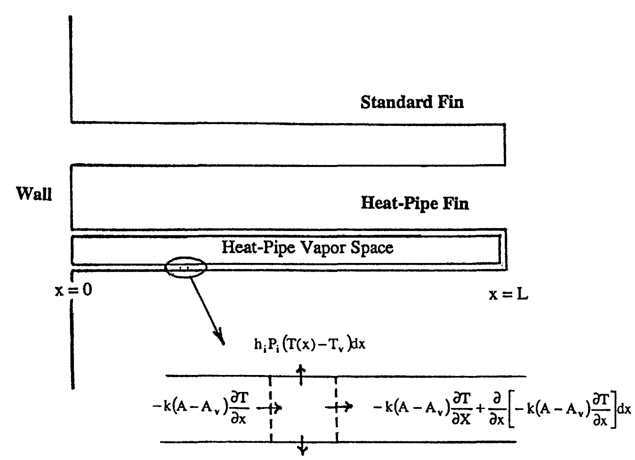

Fig. 1 Heat pipe fin control volume.

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heat pipe, P_i is the perimeter of the vapor space, and T_v is the vapor temperature inside the heat pipe.

The nondimensional temperature and length can be defined as

$$\theta = (T - T_\infty)/(T_b - T_\infty) \tag{2}$$

$$X = x/L \tag{3}$$

where L is the length of the heat pipe, and T_b is the fin base temperature. Rewriting Eq. (1) in terms of the nondimensional variables gives

$$\frac{\partial^2\theta}{\partial X^2} - a^2(m^2 + n^2)\theta = -a^2 n^2 \theta_v \tag{4}$$

where

$$m^2 = h_0 P_0 L^2/kA \tag{5}$$

$$n^2 = h_i P_i L^2/kA \tag{6}$$

$$a^2 = A/(A - A_v) \tag{7}$$

$$\theta_v = (T_v - T_\infty)/(T_b - T_\infty) \tag{8}$$

The parameters a^2, n^2, and m^2 were carefully selected to make it easy to compare the heat pipe fin's performance with a standard fin. m^2, as defined in Eq. (5), is a standard parameter used in fin design. It is the ratio of conduction resistance (L/kA) to convection resistance ($1/h_0P_0L$) for the standard fin. A similar parameter that is important in heat pipe fins, but not for standard fins, is n^2. It is the ratio of conduction resistance along the fin to convection resistance associated with the heat pipe wick and evaporation on the inside of the heat pipe fin ($1/h_iP_iL$). The last parameter, a^2, is the ratio of the total fin cross-sectional area (A) to the heat pipe wall area ($A - A_v$). Realistic values for these three parameters and their importance will be discussed later in this note.

Equation (4) can be solved for $\theta(X)$. Assuming that T_∞, T_v, h_i, and h_0 are constants, the solution is

$$\theta(X) = c_1 \exp(ZX) + c_2 \exp(-ZX) + \frac{n^2\theta_v}{(m^2 + n^2)} \tag{9}$$

where $Z = a(m^2 + n^2)^{1/2}$. The constants c_1 and c_2 are evaluated by applying the appropriate boundary conditions. Assuming that the base temperature is known and that the fin tip is insulated, gives the boundary conditions

$$\theta(0) = 1.0 \tag{10}$$

$$\frac{\partial\theta}{\partial X}(1) = 0.0 \tag{11}$$

The constants c_1 and c_2 are

$$c_1 = \frac{1 - [n^2\theta_v/(m^2 + n^2)]}{1 + \exp(2Z)}, \quad c_2 = \frac{1 - [n^2\theta_v/(m^2 + n^2)]}{1 + \exp(-2Z)} \tag{12}$$

With the constants evaluated, the temperature distribution in the heat pipe is known:

$$\theta(X) = \left[1 - \left(\frac{n^2\theta_v}{m^2 + n^2}\right)\right]\frac{\cosh[Z(X - 1)]}{\cosh(Z)} + \frac{n^2\theta_v}{m^2 + n^2} \tag{13}$$

An important fin parameter is the fin efficiency. It is defined as the heat transfer from the fin divided by the maximum heat transfer if the entire fin is maintained at the fin's base temperature. The efficiency can be found from the expression

$$\eta = \frac{\int_0^L h_0 P_0 (T - T_\infty)\,\mathrm{d}x}{h_0 P_0 L (T_b - T_\infty)} = \int_0^1 \theta\,\mathrm{d}x \tag{14}$$

Evaluating Eq. (14) for the case of the heat pipe fin gives

$$\eta_{\mathrm{HP},f} = \frac{\tanh(Z)}{Z + [n^2/(m^2 + n^2)][\tanh(Z) - Z]} \tag{15}$$

The heat pipe fin efficiency can be compared with the efficiency of a standard, constant-area fin with the same boundary conditions and geometry. The efficiency of the standard fin is

$$\eta_f = \tanh m/m \tag{16}$$

The preceding derivation requires additional clarification. It was assumed that the convection coefficient inside the heat pipe was uniform along the heat pipe. This is a reasonable assumption. Energy transport from the wall to the vapor occurs through the liquid-saturated wick and then from the wick surface to the vapor. This energy transport can be represented using an electrical analogy as energy transport through two thermal resistors. The thermal resistance of the wick is much larger than the thermal resistance because of evaporation or condensation.[4] Thus, h_i can be approximated by examining conduction through the wick, or

$$h_i = k_{\mathrm{eff}}/t \tag{17}$$

where t is the thickness of the heat pipe wick, and k_{eff} is the effective thermal conductivity of the wick. Assuming the liquid is uniformly distributed along a wick of uniform thickness, this thermal resistance will also be uniform along the length of the heat pipe.

The nondimensional vapor temperature, θ_v, deserves some discussion. The nondimensional vapor temperature in the heat pipe is a parameter in the heat pipe fin axial temperature distribution [Eqs. (9), (12), and (13)]. This temperature can be found by considering the conservation of energy as applied to the vapor in the heat pipe. After some manipulation, and referring back to Eq. (14), it can be seen that the nondimensional vapor temperature is equal to the heat pipe fin efficiency:

$$\int_0^L h_i P_i (T - T_v)\,\mathrm{d}x = 0, \qquad \int_0^1 \theta\,\mathrm{d}X = \theta_v = \eta_{\mathrm{HP},f} \tag{18}$$

This was used in the efficiency derivation to eliminate θ_v from the efficiency equation.

Now that an expression for the efficiency of the heat pipe fin has been obtained, it can be used to access the usefulness of this type of fin.

Comparison of the Heat Pipe Fin with a Standard Fin

Equations (15) and (16) can be compared to determine when the heat pipe fin is more efficient than a standard fin. Figures 2–4 graphically represent Eqs. (15) and (16) for three different values of the parameter a. From the figures, it can be seen that the heat pipe fin will be more efficient than the standard fin if a is small (thick heat pipe wall), or if n is large (long fin, low thermal conductivity fin, or large h_i, which would occur for a thin wick design or high thermal conductivity wick). In Fig. 2, ($a = 2$), the heat pipe fin is more efficient than the standard fin for values of n greater than about 7. From Fig. 3, n must

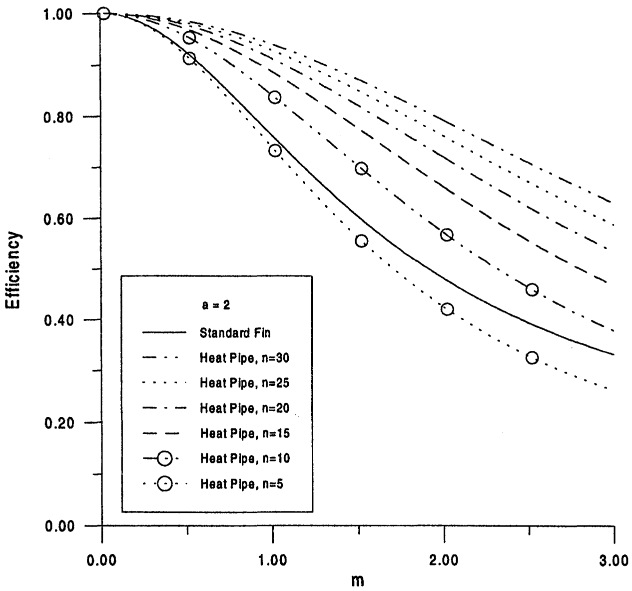

Fig. 2 Comparison of standard fin and heat pipe fin, *a* = 2.

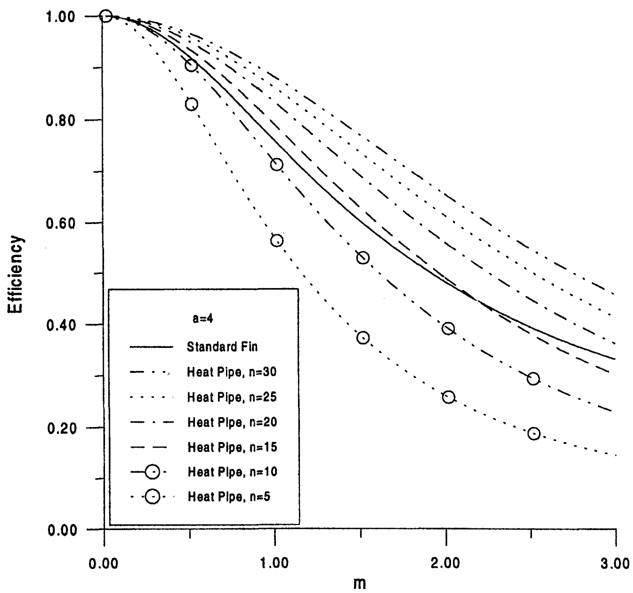

Fig. 3 Comparison of standard fin and heat pipe fin, *a* = 4.

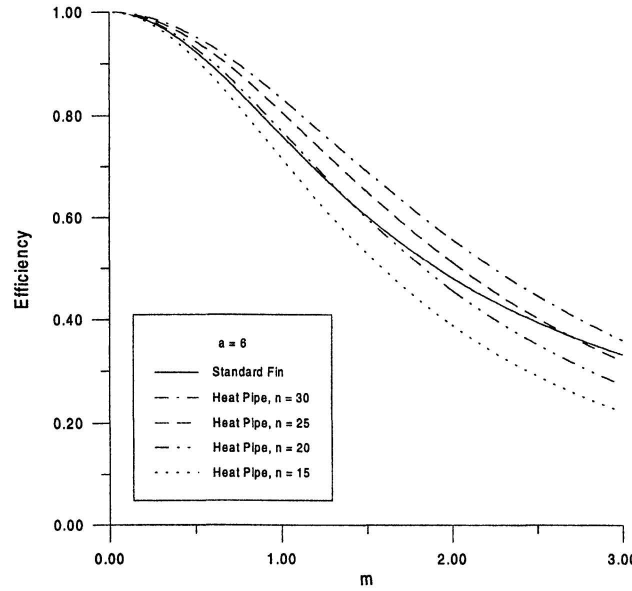

Fig. 4 Comparison of standard fin and heat pipe fin, *a* = 6.

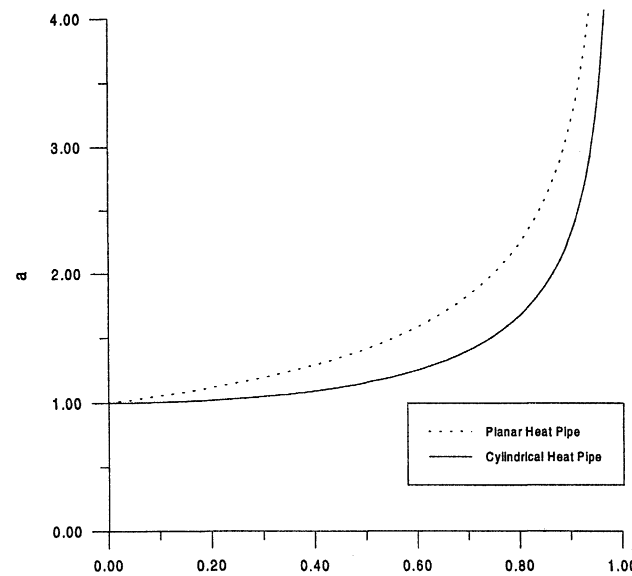

Fig. 5 Typical values for the parameter *a*.

be larger than about 15 if $a = 4$. When $a = 6$, n must be greater than 20 to 25 for the heat pipe fin to be more efficient, see Fig 4.

As discussed earlier, two important parameters associated with heat pipe fin design are a and n, which are defined by Eqs. (5) and (6). For cylindrical and planar fins, Eq. (6) can be rewritten as

$$\text{(cylindrical)}\ a = \frac{1}{(1 - R^2)^{1/2}}$$

$$\text{(planar)}\ a = \frac{1}{(1 - R)^{1/2}} \tag{19}$$

$$R = r_v / r$$

where r_v is the half thickness or radius of the vapor space, and r is the half thickness or radius of the fin. Equation (19) is graphed in Fig. 5. As R approached 1.0 (or as the wall thickness becomes thinner), the parameter a becomes large. As mentioned earlier, small values of a are better for heat pipe fin efficiency. As long as R is less than 0.75, a will be less than 2.0. Thicker walls will result in values of a between 1.0 and 2.0.

The parameter a regulates the rate of conduction from an object into the base of the heat pipe fin. As the heat pipe wall becomes thicker, a decreases. The thicker wall enhances conduction from the object to the heat pipe evaporator region. Thin heat pipe walls are desirable in the heat pipe condenser region. For the fins studied in this project, thick walls are desirable in the evaporator to conduct energy along the evaporator. Another method of enhancing conduction in the evaporator is to embed the evaporator into the object to be cooled. This would increase the evaporator surface area subjected to the high object base temperature. This condition was not studied as part of this analysis.

As the heat pipe wall becomes thicker, the vapor space area decreases. There is a limit to the amount that the vapor space area can be decreased. For example, if the vapor space becomes too small, the heat pipe sonic limit will be reached. An optimum value for a must exist; however, as can be seen from Fig. 5, small values of area ratio a can be obtained with a wide range of wall thickness values.

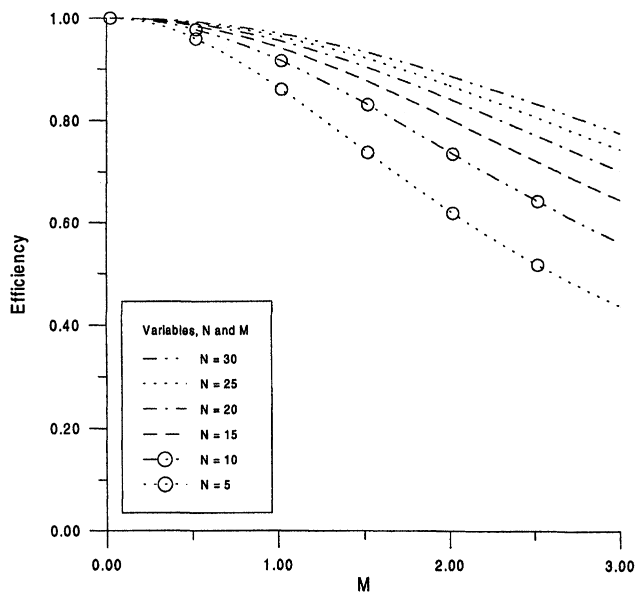

Fig. 6 Heat pipe fin efficiency as a function of *M* and *N*.

The other important parameter is n. In examining Eq. (5), n is very similar to m, defined in Eq. (4). The only difference between the two parameters is the convection coefficient and perimeter values. Looking at the ratio of the two terms

$$\frac{n}{m} = \left(\frac{h_i P_i}{h_0 P_0}\right)^{1/2} \tag{20}$$

Dunn and Reay[4] discuss the relative magnitudes of convection thermal resistance in heat pipes. They show that the resistance to convection on the outside of the heat pipe can range from 10 to 10^3°C/W, whereas the resistance through the heat pipe wick is on the order of 10°C/W. Thus, n will range in magnitude from m to $10m$. From Eq. (6), it can be seen that n will be large for heat pipe fins with large aspect ratios (L^2/A) or low thermal conductivity.

The variables a, n, and m were selected for convenience in comparing the standard fin with the heat pipe fin. A more convenient set of variables that apply only to the heat pipe fin are

$$M = am, \qquad N = an \tag{21}$$

Figure 6 shows how heat pipe fin efficiency varies with M and N. Figure 6 can be used as a design tool. For a particular heat pipe fin, the parameters N and M can be calculated. Then, using Fig. 6, the thermal efficiency of the fin can be found. As defined earlier in Eq. (14), the thermal efficiency is the fraction of the maximum heat transfer from the fin. The maximum heat transfer assumes the entire fin is at the fin base temperature. Unfortunately, N and M cannot be used to compare the heat pipe fin directly with a geometrically similar standard fin. The parameters a, n, and m are required for that comparison.

Conclusions

This paper has dealt with the comparison between a heat pipe fin and a standard fin. An analytical expression for the heat pipe fin temperature distribution and efficiency were obtained. The results show that heat pipe fins may not always be more efficient than standard fins; however, they usually are. The heat pipe fins are beneficial if weight is important, if the fins must be made from low thermal conductivity materials, or if high aspect ratio fins are needed.

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Comparison of Effective Thermal Conductivity and Contact Conductance of Fibrous Composites

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Introduction

FIBER-REINFORCED plastics are increasingly used in structural and thermal applications, necessitating an understanding of the thermophysical and mechanical properties. The strength and fatigue properties of these materials are reasonably well characterized and documented; however, limited information is available on the thermal properties, such as effective thermal conductivity. A review of published literature[1] indicates that the type of fiber and matrix, fiber volume fraction, and fiber orientation play critical roles in the effective thermal conductivity of the composite.

The objective of this experimental investigation is to present a comparison of the effective thermal conductivity and contact conductance for several different cured fibrous nonmetallic composites. Furthermore, this Note also discusses several previously conducted experimental studies and makes a comparison of these data with the results obtained in the present study.

The most prominent factors that affect the thermal conductivity of the composite materials are the thermal conductivity of the matrix and the fiber, and the fiber volume fraction.[2] There are a variety of polymers that are used as matrix materials for fiber-reinforced composites, including unsaturated polyesters, epoxies, polyethylene, and polycarbonates. Another critical factor to be considered for carbon fiber materials is the structure and type of carbon fiber; those based on a pitch precursor and those based on a polyacrylonitrile (PAN) precursor. These different precursors exhibit different thermal conductivity values. Further, graphite fibers, unlike carbon and glass fibers, exhibit a microstructural feature or texture that causes the transverse and longitudinal thermal conductivity to be anisotropic.

Other factors to be considered are the spatial characteristics and the interfacial thermal resistance between the fiber and the surrounding material. The spatial characteristics include the size, shape, and spacing of the fibers. Invariably, each fiber is in contact with the matrix, another fiber, a pore, or some combination thereof. Each of these boundaries will exhibit a resis-

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